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2 013 791 13C40 94B27

Hansen, Johan P. (DK-ARHS-MI)

Linkage and codes on complete intersections. (English. English summary)

Appl. Algebra Engrg. Comm. Comput. **14** (2003), *no.* 3, 175–185.

From the summary: "This note is meant to be an introduction to cohomological methods and their use in the theory of error-correcting codes."

Let $\mathbb{F}_q$ be the finite field of q elements and let $X = \{P_1, \ldots, P_n\}$ be a set of n $\mathbb{F}_q$-rational points in the projective space $\mathbb{P}^l(\mathbb{F}_q)$. Let $S = \mathbb{F}_q[x_0, \ldots, x_l]$ and $S_{(d)}$ be the set of homogeneous polynomials of degree d, for any natural number d. Let $f_0 \in S_{(d)}$ be such $(f/f_0)(P) \neq 0$ for all $P \in X$. The linear code $C_d(X)$ is defined as the image of $S_{(d)}$ under the evaluation map $f \mapsto ((f/f_0)P_1, \ldots, (f/f_0)P_n)$. Thus it is a subspace of $\mathbb{F}_q^n$, the dimension of which is obtained from the Hilbert function of S/I, where I is the ideal of X, and which can be read in the resolution of S/I. (This code belongs to the family of Reed-Muller codes.)

In the case when X is a complete intersection defined by a regular sequence of homogeneous polynomials, the resolution of S/I is given by a Koszul complex and thus gives an explicit formula for $\dim C_d(X)$. In some cases, a lower bound for the minimal distance of $C_d(X)$ is obtained through linkage. The following is proved.

Theorem. Let $X \subset \mathbb{P}^2(\mathbb{F}_q)$ be a complete intersection of two curves of degree d_1, d_2, with $|X| = d_1 \cdot d_2$ and $d_i \geq 3$. Let $d > d_i - 3$ for $i = 1, 2$ and let $d \leq d_1 + d_2 - 3$. Then the evaluation code $C_d(X)$ has minimal distance $\geq d_1 + d_2 - d - 1$.

The author considers a subset V_1 of X with $|V_1| > d_1 \cdot d_2 - (d_1 + d_2 - d - 3)$. He takes advantage of the fact that V_1 and $V_2 = X \setminus V_1$ are perfect varieties linked by the complete intersection X, and he makes use of the projective resolution of the structure sheaf of V_1 described by Peskine and Szpiro in terms of the structure sheaves of V_2 and X. He also takes advantage of the fact that V_2 does not have too many points to conclude that $\dim C_d(X) = \dim C_d(V_1)$. The result is as follows.

In the case when X is an affine plane contained in $\mathbb{P}^2(\mathbb{F}_q)$, thus a complete intersection with defining ideal $(x_2^q - x_0^{q-1}x_2, x_1^q - x_0^{q-1}x_1)$, he recovers the lower bound for the minimal distance of $C_d(X)$ obtained previously by other methods. *Anne-Marie Simon* (B-ULB-AL)

2003j:14029 14G50 14M25 94B27

Hansen, Johan P. (DK-ARHS)

Toric varieties Hirzebruch surfaces and error-correcting codes. (English. English summary)

Appl. Algebra Engrg. Comm. Comput. **13** (2002), *no.* 4, 289–300.

Let P be a convex polygon in $\mathbf{R}^2$ whose vertices are in $\mathbf{Z}^2$. Let q be a prime power and let ξ be a primitive element of $\mathbf{F}_q$. For $0 \leq i, j \leq q-2$ let $P_{ij} = (\xi^i, \xi^j)$ in $\mathbf{F}_q^* \times \mathbf{F}_q^*$. For each $m = (m_1, m_2) \in P \cap \mathbf{Z}^2$, let $e(m)(P_{ij}) = (\xi^i)^{m_1}(\xi^j)^{m_2}$. The toric code C_P associated to P is by definition the linear code of block length $n = (q-1)^2$ spanned by the vectors in $\{(e(m)(P_{ij}))_{0\leq i,j\leq q-2} \colon m \in P \cap \mathbf{Z}^2\}$. This definition generalizes the construction of Reed-Solomon codes as evaluation codes and was introduced by the author [in *Coding theory, cryptography and related areas (Guanajuato, 1998)*, 132–142, Springer, Berlin, 2000; MR1749454 (2000m:94036)].

The toric surface associated to the normal fan of the polytope P underlies the construction and properties of these codes. In particular, the monomials $e(m)$ which are evaluated to produce the generating codewords can be interpreted as sections of a certain line bundle on that surface, and intersection theory on the toric surface can be used to derive information about the minimum distance of toric codes. The codes from the two families of triangles and rectangles considered in the article cited above are taken up again and codes from polygons P corresponding to Hirzebruch surfaces are studied in detail. The exact minimum distance is determined by exhibiting codewords of weight equal to a lower bound derived from intersection theory.

John B. Little (1-HLYX)

2004c:11107 11G20 94B27 94B35

Hansen, Johan P. (DK-ARHS-MI)

Dependent rational points on curves over finite fields— Lefschetz theorems and exponential sums. (English. English summary)

International Workshop on Coding and Cryptography (*Paris*, 2001), 13 *pp.* (*electronic*), *Electron. Notes Discrete Math.*, 6, *Elsevier*, *Amsterdam*, 2001.

Let C be a curve over $\mathbf{F}_q$. The authors estimate the probability that τ randomly chosen $\mathbf{F}_q$-rational points on C do not impose τ linearly independent conditions on functions from a given τ-dimensional space of rational functions on C. It is shown that the above probability tends to $1/q$ when $q \to \infty$. These result has direct applications to some coding theory problems. *Igor E. Shparlinski* (5-MCQR-CP)

2002m:68046 68Q17 68Q05 68W30

Frandsen, Gudmund Skovbjerg (DK-ARHS-BR);
Hansen, Johan P. (DK-ARHS);
Miltersen, Peter Bro (DK-ARHS-BR)

Lower bounds for dynamic algebraic problems. (English. English summary)

Inform. and Comput. **171** (2001), *no.* 2, 333–349.

J. H. Reif and S. R. Tate [J. Algorithms **22** (1997), no. 2, 347–371; MR1428342 (98i:68126)] introduced the following problem of incrementally evaluating rational functions $f_1, \ldots, f_n$. The system is allowed to do some preprocessing on an initial input vector $x_1, \ldots, x_n$. Then, the algorithm is required to efficiently answer on-line requests of the form: "change the ith input to value v" and "return the value of $f_j(x)$".

The paper presents a general lower bound technique for addressing these dynamic evaluation problems, based on certain injectivity properties of maps. The proof relies on elementary dimension arguments of algebraic geometry. The lower bound is first proved for history dependent algebraic computation trees and then extended to a wide range of computational models.

The technique yields optimal lower bounds of order n for classical computational problems like matrix multiplication, polynomial evaluation and discriminant. Lower bounds of order n are also obtained for determinant and matrix inverse, whereas for convolution and elementary symmetric functions the technique yields lower bounds of order $\sqrt{n}$. For the latter problems optimality remains open.

Finally, the authors use the lower bound for depth-two superconcentrators due to J. Radhakrishnan and A. Ta-Shma [SIAM J. Discrete Math. **13** (2000), no. 1, 2–24 (electronic); MR1737930 (2001a:94055)] for proving an $\Omega(\log^2 n/ \log\log n)$ lower bound for the dynamic evaluation of the discrete Fourier transform. Here, there remains an exponential gap between the best known upper and lower bounds.

Peter Bürgisser (D-PDRB-C)

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2000m:94036 94B05 14M25 94B27

Hansen, Johan P. (DK-ARHS-MI)

Toric surfaces and error-correcting codes. (English. English summary)

Coding theory, cryptography and related areas (*Guanajuato*, 1998), 132–142, *Springer*, *Berlin*, 2000.

Summary: "From an integral convex polytope in $\mathbf{R}^2$ we give an explicit description of an error-correcting code over the finite field $\mathbf{F}_q$ of length $(q-1)^2$. The codes are obtained from toric surfaces and the results are proved using the cohomology and intersection theory of such surfaces. The parameters of three such families of toric codes are determined."

{For the entire collection see 2000k:94001}

1 734 065 68W30

Frandsen, Gudmund Skovbjerg (DK-ARHS-BRI)**;**
Hansen, Johan P. (DK-ARHS)**;**
Miltersen, Peter Bro (DK-ARHS-BRI)

Lower bounds for dynamic algebraic problems. (English. English summary)

STACS 99 (*Trier*), 362–372, *Lecture Notes in Comput. Sci.*, 1563, *Springer*, *Berlin*, 1999.

99h:94066 94B27

Hansen, Johan P. (DK-ARHS)**;**
Jensen, Helge Elbrønd (DK-TUD)**; Kötter, Ralf** (1-IL-S)

Determination of error values for algebraic-geometry codes and the Forney formula. (English. English summary)

IEEE Trans. Inform. Theory **44** (1998), *no.* 5, 1881–1886.

Summary: "In the decoding of one-point algebraic-geometry codes with one defining equation in two variables we generalize the Forney formula for the Bose-Chaudhuri-Hocquenghem (BCH) codes."

95e:94055 94B27 11T71 13D40

Hansen, Johan P. (DK-ARHS-MI)

Points in uniform position and maximum distance separable codes. (English. English summary)

Zero-dimensional schemes (*Ravello*, 1992), 205–211, *de Gruyter*, *Berlin*, 1994.

This paper presents a general construction of linear error-correcting codes from families of points in projective space. The codewords are values of rational functions obtained from all homogeneous polynomials of given degree on a set of points. The parameters of the codes are calculated from values of Hilbert functions of the corresponding family. Conditions for the construction to yield MDS codes are determined. This depends essentially on the number of points in uniform position. The points are said to be in uniform position if subsets of points of the same size have the same Hilbert function.

{For the entire collection see 95c:14001}

Simon N. Litsyn (Ramat-Aviv)

94h:14024 14H05 11G20 14G05 14G15

Hansen, Johan P. (DK-ARHS-MI)**;**
Pedersen, Jens Peter (DK-TUD)

Automorphism groups of Ree type, Deligne-Lusztig curves and function fields.

J. Reine Angew. Math. **440** (1993), 99–109.

The authors prove the following uniqueness realization result: If F is a function field over $\mathbf{F}_q$ of genus $\frac{3}{2}q_0(q-1)(q+q_0+1)$, where $q=3^{2s+1}$ and $q_0=3^s$, such that all automorphisms in $G=\mathrm{Aut}(F|\overline{\mathbf{F}}_q)$ are $\mathbf{F}_q$-rational, and G is a Ree group of order $q^3(q-1)(q^3+1)$ acting as a permutation group on the set of q^3+1 rational points, then F is $\mathbf{F}_q$-isomorphic to $\mathbf{F}_q(x,y_1,y_2)$, where $y_1^q-y_1=x^{q_0}(x^q-x)$ and $(y_2^q-y_2)=x^{2q_0}(x^q-x)$.

These function fields are interesting for a number of reasons. First of all, a Ree group is one of the three cases of simple groups where the associated Deligne-Lusztig variety [see P. Deligne and G. Lusztig, Ann. of Math. (2) **103** (1976), no. 1, 103–161; MR0393266 (52 #14076); exposition, J.-P. Serre; MR0435240 (55 #8200)] is an irreducible curve (the other cases being Suzuki groups and projective special linear groups). Also, the number of rational points is maximal for the given genus, and for suitable constant field extensions the bound of Hasse-Weil is attained.

The proof follows from the study of the ramification of the extensions $F|F^{G_i}$, where the G_i are ramification groups of a rational point P_∞ [see also H. Stichtenoth, Arch. Math. (Basel) **24** (1973), 615–631; MR0404265 (53 #8068)]. *Paulo Viana* (Rio de Janeiro)

94e:94024 94B27 11T71 14H52

Hansen, Johan P. (DK-ARHS-MI)

Deligne-Lusztig varieties and group codes. (English. English summary)

Coding theory and algebraic geometry (*Luminy*, 1991), 63–81, *Lecture Notes in Math.*, 1518, *Springer*, *Berlin*, 1992.

Algebraic-geometric codes are constructed using the Deligne-Lusztig varieties associated to a connected reductive algebraic group defined over a finite field $\mathbf{F}_q$ with Frobenius map F. The Deligne-Lusztig varieties used have in some cases many $\mathbf{F}_q$-rational points which implies that the codes have a large word length.

{For the entire collection see 93d:11002}

N. L. Manev (BG-AOS-IMI)

96e:94023 94B27 14G15 14H99

Hansen, Johan P. (DK-ARHS-MI);
Stichtenoth, Henning (D-ESSN)

Group codes on certain algebraic curves with many rational points. (English. English summary)

Appl. Algebra Engrg. Comm. Comput. **1** (1990), *no.* 1, 67–77.

Good codes constructed from algebraic curves have a large number of rational points. It is of considerable interest, therefore, to construct families of curves for which the number of rational points is the largest possible for the genus of each curve in the family. In this paper the authors study curves S_q and the corresponding codes over $\mathbf{F}_q$, where $q = 2q_0^2,\ q_0 = 2^n$, the genus of S_q is $g = q_0(q-1)$ and the number of rational points is $N_1 = q^2+1$, the maximum attainable. It should be noted that N_1 is strictly less than the Hasse-Weil bound $q+1+2g\sqrt{q}$. The affine equation of S_q, which has the Suzuki simple group $\mathbf{Sz}_q$ as automorphism group, is $z^q + z = y^{q_0}(y^q + y)$.

J. W. P. Hirschfeld (4-SUSX)

89f:11164 11T71 94B35

Hansen, Johan P. (DK-ARHS)

Codes on the Klein quartic, ideals, and decoding.

IEEE Trans. Inform. Theory **33** (1987), *no.* 6, 923–925.

The author constructs codes over $F = \mathrm{GF}(2^3)$ using Goppa's link between coding theory and the theory of algebraic curves over a finite field. Five codes, based on the same curve, are discussed (two a bit more thoroughly than the others); their (n, k, d) parameters are $(21, 4, 15)$, $(21, 7, 12)$, $(21, 10, 9)^*$, $(21, 13, 6)$, and $(21, 16, 3)^*$. An important innovation is the exploitation of symmetries of the curve. The codes "turn out" to "be" left ideals in a group algebra $F[G]$ and this in turn leads to a decoding strategy based on the algebra's idempotents (this relates to work of Dangaard and Landrock).

More specifically, the author starts with the Klein quadric C. This is the nonsingular, projective, plane curve defined by $x^3y + y^3z + z^3x = 0$. The Frobenius group G (a group of order 21 given by generators and relations $G = \langle A, B \colon A^7 = 1,\ B^3 = 1,\ B^{-1}AB = A^4\rangle$) operates, over F, on this curve. C has 24 F-rational points falling into two G-orbits. One of these has three points (christened Q_0, Q_1, and Q_2); the other orbit has 21 points which can thus be identified with the elements of G.

The five codes arise from the G-invariant divisors $D = m(Q_0 + Q_1 + Q_2)$ for $2 \le m \le 6$. Following theory, the codes are the images of $L(D)$ in $F^{21} \cong F[G]$ obtained by evaluating the functions in $L(D)$ on the points just identified with G. (Here $F[G]$ denotes the group algebra with coefficients F of the group G.) The group G acts as a group of symmetries on each of the codes. The codes themselves are shown to be left ideals of $L(G)$ and a decoding scheme based on this fact is described. For the $(21, 16, 3)$ code, which corrects one error, the author has implemented a decoding algorithm. *A. T. Vasquez* (1-CUNY2)

87k:14005 14C17 14E22 14N05

Hansen, Johan P. (DK-ARHS); **Vyrdal, Simon** (DK-ARHS)

Double points of compositions of projections.

Math. Scand. **58** (1986), *no.* 1, 119–124.

Let $X \subseteq \mathbf{P}^M$ be a variety, $L \subseteq L' \subseteq \mathbf{P}^M$ linear spaces such that L' does not meet X. Then one can consider the double point loci and ramification loci of the linear projections π_L and $\pi_{L'}$ of X. The authors give a scheme-theoretic relation between the double point locus of π_L and the double point locus and ramification locus of $\pi_{L'}$. In the case $\dim L' = \dim L + 1$ and L and L' are generic, they give the cycle-theoretic version of this relation. From this they deduce a formula giving the double point class of π_L as the difference between a hyperplane section of the double point class of $\pi_{L'}$ and the ramification class of $\pi_{L'}$. This formula was first obtained by K. W. Johnson [Acta Math. **140** (1978), no. 1-2, 49–74; MR0463161 (57 #3120)].

Ragni Piene (N-OSLO)

87e:14013 14E22 14E20

Hansen, Johan P. (DK-ARHS)

Higher order singularities of morphisms to projective space.

Proc. Amer. Math. Soc. **97** (1986), *no.* 2, 226–232.

Let $f\colon X \to \mathbf{P}^n$ be a finite morphism from the algebraic variety X to $\mathbf{P}^n$ over $\mathbf{C}$. A point $x \in X$ is called a qth order singularity (for f) if $\dim_{\mathbf{C}}(\mathcal{O}_{X,x}/f^* m_{f(x)} + m_x^{q+1}) \geq q+1$. Denote by S_q the closed subset of X consisting of all qth order singular points for f, and define $\sigma_q = \dim(X) - q(n - \dim(X) + 1)$. Based on R. Schwarzenberger's notion of generalized secant sheaves and the Fulton-Hansen connectedness theorem, the author proves the following results. Theorem 1: If X is normal and irreducible and such that $S^q = \varnothing$ for some q with $\sigma_q \geq 0$, then X is simply connected. Theorem 2: If X is irreducible and $\# f^{-1}(f(x)) \geq 2$ for every $x \in X$, then S^q is nonempty and has an irreducible component of dimension $\geq \sigma_q$ for all q with $\sigma_q \geq 0$.

L. Bădescu (R-NISTC)

86b:14010 14E22

Hansen, Johan P. (DK-ARHS)

Double-points of compositions.

J. Reine Angew. Math. **352** (1984), 71–80.

Let $f\colon X\to Y$ and $g\colon Y\to V$ be morphisms of quasiprojective schemes over an algebraically closed field. Let $p\colon X\tilde{\times}X\to X\times X$ denote the blowing up of the diagonal in $X\times X$. Let $Z(f)$ denote the double point scheme of f, and let $R(f)$ denote the ramification locus of f. Let h denote the restriction of $(f\times f)\circ p$ to $Z(g\circ f)$ (with image in $Y\times_V Y$). Then the main theorem of this paper is the following equality: $h^{-1}(\Delta_y)=\mathrm{Im}[Z(f)\coprod R(g\circ f)]$. Here Im denotes the image as defined by B. Teissier [Real and complex singularities (Oslo, 1976), 565–678, Sijthoff & Noordhoff, Alphen aan den Rijn, 1977; MR0568901 (58 #27964)], and is a useful device for compacting the statement and the proof of the theorem. The proofs themselves are mostly formal. The theorem is applied in the case that $g\colon Y\to V$ is an ample vector bundle, as for example when it is a linear projection of projective spaces. A more precise formula is given in the case that all schemes are smooth and the ample bundle is a line bundle. In the cases of linear projections, the author recovers theorems previously proved by K. W. Johnson [Acta Math. 140 (1978), no. 1–2, 49–74; MR0463161 (57 #3120)]. *Allen B. Altman* (Great Barrington, MA)

85d:14071 14M15

Hansen, Johan

A connectedness theorem for flagmanifolds and Grassmannians.

Amer. J. Math. **105** (1983), *no.* 3, 633–639.

The central result of this article is the following: Let F be any flagmanifold of flags in $\mathbf{P}^m$. If $f\colon X\to F\times F$ is any morphism from a complete irreducible variety X, then (1) $f^{-1}(\Delta_F)$ is nonempty if $\operatorname{codim}(f(X),F\times F)\le m$, and (2) $f^{-1}(\Delta_F)$ is connected if $\operatorname{codim}(f(X),F\times F)<m$. The proof is easily reduced to the case of the full flagmanifold F_m and then proved by induction on k, where $F_k=F(0,1,\cdots,k;m)$, the case $k=0$ having been proved by the author and W. Fulton [Ann. of Math. (2) 110 (1979), no. 1, 159–166; MR0541334 (82i:14010)]. *Allen B. Altman* (Great Barrington, MA)

83g:14001 14A25

Hansen, Johan P.

Connectedness theorems in algebraic geometry.

18*th Scandinavian Congress of Mathematicians* (*Aarhus*, 1980), *pp.* 336–346, *Progr. Math.*, 11, *Birkhäuser*, *Boston*, *Mass.*, 1981.

This is a short account, without proofs, of the Fulton-Hansen connectedness theorem and some of its improvements and recent applications.

{For the entire collection see 82i:00012}

Knud Lønsted (Copenhagen)

82i:14010 14E20 14E22

Fulton, William; Hansen, Johan

A connectedness theorem for projective varieties, with applications to intersections and singularities of mappings.

Ann. of Math. (2) **110** (1979), *no.* 1, 159–166.

The central result of the paper is the following connectedness theorem. Let $f\colon X \to \mathbf{P}^m \times \mathbf{P}^m$ be a morphism of an irreducible variety into the product of projective spaces with $\dim f(X) > m$; then the inverse image of the diagonal is connected. This very important result has many interesting corollaries. For example, if X is an irreducible subvariety of $\mathbf{P}^m$ of dimension $n > m/2$ then X has no nontrivial étale coverings. This result was previously known for local complete intersection varieties in characteristic zero or Cohen-Macaulay varieties in positive characteristic. Or, a singular irreducible variety of dimension n with only normal crossings cannot be embedded in $\mathbf{P}^{2n-1}$. Recently, many other interesting applications of the connectedness theorem have been found [the first author and R. Lazarsfeld, *Algebraic geometry* (Chicago, Ill., 1980), pp. 26–92, Lecture Notes in Math., 862, Springer, Berlin, 1981]. The most striking of them is the proof (by the first author) of Zariski's celebrated conjecture on the fundamental group of the complement to a nodal plane curve. A certain generalization (to a topological situation) of the main theorem was obtained by P. Deligne [*Bourbaki Seminar, Vol. 1979/80*, pp. 1–10, Lecture Notes in Math., 842, Springer, Berlin, 1980]. *I. Dolgachev* (Bonn)